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Similarity analysis of planar and axisymmetric turbulent synthetic jets

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ABSTRACT

Data on synthetic jets suggest that planar and axisymmetric turbulent synthetic jets exhibit self-similarity in the far field. A similarity analysis, similar to that of continuous turbulent jet, for both these two-dimensional cases, has been undertaken in this paper. Important differences between synthetic and continuous jets arise because of a larger spread rate in the case of synthetic jets. The analysis predicts the same streamwise variation of velocity and spread rate with synthetic jets as the corresponding continuous jets. It is argued that to first order, the momentum flux with contribution from both mean and fluctuating velocity should be conserved in the self-similar region, but with a value less than that supplied at the source. The predictions of similarity analysis are supported by the experimental data.

1. Introduction

Synthetic jets are produced by periodic ejection and suction of fluid from an orifice induced by movement of a diaphragm inside a cavity [1,2], among other ways. From the flow visualization images of Verma and Agrawal [3] the following sequence of jet formation can be gathered. At low Strouhal numbers, a train of vortices forms due to movement of the diaphragm; these vortices travel away from the orifice if the Strouhal number is larger than a critical value (jet formation criteria) [4]. The distance between the vortices reduces with streamwise coordinate at higher Strouhal numbers, and the vortices eventually merge to form a laminar jet. With a further increase in Strouhal number, an instability appears in the near field which breaks down the laminar structure of the jet, in to first a transitional jet, and then a fully turbulent jet [5]. The synthetic jets are expected to find important engineering applications like in boundary layer separation control, jet vectoring, heat transfer enhancement, better mixing of fuel in the engine combustion chamber, creation of local turbulence, and vehicle propulsion (see example Refs. [6–9], among others).

The synthetic jets are zero-net-mass-flux flows because the mass ejected from the cavity during the ejection stroke is exactly balanced by the mass sucked during suction. Although no net mass is added at the orifice, there is a net addition of momentum due to the movement of the diaphragm, and a jet forms due to entrainment of ambient fluid as this momentum pulse travels downstream. On the other hand, conventional jets are formed by the addition of both mass and momentum at the orifice [10–12]. Unlike synthetic jets, the momentum flux in conventional jets remains conserved [1]. Due to the difference in formation between the two jets, the near-field behaviour is expected to be different. For example, there is no potential core in synthetic jets as opposed to conventional jets [5,13].

However, both these types of jets are known to exhibit self-similarity beyond a certain distance from the orifice; this distance is comparable between synthetic and continuous jets. For example, the experimental data of Mallinson et al. [14] suggest that an axisymmetric turbulent synthetic jet exhibits self-similarity at about 10 times the orifice diameter. Cater and Soria [5] report a distance of 15 d (where d is the orifice width or diameter) based on mean velocity and 25 d based on Reynolds stress. Verma [13] found this distance to be about 10 d based on the decay of centerline velocity. Smith and Swift [15] reported self-similarity beyond 13 d for a planar turbulent synthetic jet. Fugal et al. [16] argue that the displacement amplitude is the more relevant length scale for determining the self-similar distance. The self-similarity is mostly deduced by the good collapse of mean velocities or Reynolds stresses on to a single curve. The primary simplification that results from self-similarity is that a single velocity scale (mean streamwise velocity at the centerline) and length scale (jet half-width), are adequate to non-dimensionalize the flow parameters [17–19].

In this paper, a similarity analysis similar to that of continuous jets is undertaken, with experimental observations reported in the literature serving as a guide for analysis. Specifically, we note that the spread rate of synthetic jets is larger than their continuous counterparts (this is evident from the flow visualization images [5,13] and Table 2 presented later), and that the momentum flux of synthetic jet based on just the mean velocity is not constant [1]. This analysis leads to the prediction of streamwise variation of velocity and length scales, which can be used for benchmarking subsequent datasets. The implications of the analysis are compared with those of continuous jets. To the best of authors' knowledge, this is the first attempt of performing similarity analysis on synthetic jets.

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Nomenclature

c	spread rate of the jet ($= \mathrm{d}l/\mathrm{d}x$) (–)	$\overline{uv}$	Reynolds stresses (m^2/s^2)
c_2	constant (–)	$\overline{v^2}$	Reynolds stresses (m^2/s^2)
d	diameter or width of the orifice (m)	U	time-averaged streamwise velocity (m/s)
l	jet half-width (defined as $U(l)/U_s = 0.5$) (m)	U_s	time-averaged centerline velocity (m/s)
L	distance from the orifice (m)	V	time-averaged cross-stream velocity (m/s)
M	momentum per unit depth per unit time (kg/s^2)	x	coordinate along the axis (m)
P	time-averaged pressure (also scale for pressure) (Pa)	y	cross-stream coordinate for planar case (m)
r	cross-stream coordinate for axisymmetric case (m)	ν	kinematic viscosity of the fluid (m^2/s)
Re_l	Reynolds number (= ul/ν) (–)	ρ	density of the fluid (kg/m^3)
u	fluctuating velocity scale (m/s)	ξ	non-dimensional cross-stream coordinate ($= r/l$ for axisymmetric case, $= x/l$ for planar case) (–)
$\overline{u^2}$	Reynolds stresses (m^2/s^2)		

2. Derivation of equations

In this section, we follow the approach of Tennekes and Lumley [17] to derive the governing equations of synthetic jets in the self-similar region. The standard Reynolds decomposition for velocities and pressure has been applied in the present work; triple decomposition involving time-averaged, phase averaged and turbulent fluctuations is not required in the self-similar regime of synthetic jets, although such a decomposition may be useful near the orifice. The symbols introduced have been defined in the Nomenclature section.

2.1. Planar synthetic jet

The time-averaged streamwise momentum equation can be written as [17]

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - \frac{\partial \overline{u^2}}{\partial x} - \frac{\partial \overline{uv}}{\partial y}, \quad (1)$$

where, P is pressure, ν is kinematic viscosity and ρ is density. Estimating the order of magnitude of the various terms, we have

$$U\frac{\partial U}{\partial x} \sim \frac{U_s^2}{L} = \left[\frac{l}{L}\left(\frac{U_s}{u}\right)^2\right]\frac{u^2}{l} \quad (2)$$

$$\frac{\partial U}{\partial y} \sim \frac{U_s l}{L}\frac{U_s}{l} = \left[\frac{l}{L}\left(\frac{U_s}{u}\right)^2\right]\frac{u^2}{l} \quad (3)$$

$$\frac{1}{\rho}\frac{\partial P}{\partial x} \sim \frac{P}{\rho L} = ? \quad (4)$$

$$\nu\frac{\partial^2 U}{\partial x^2} \sim \frac{\nu U_s}{L^2} = \left[\frac{\nu}{ul}\frac{U_s l^2}{uL^2}\right]\frac{u^2}{l} \quad (5)$$

$$\nu\frac{\partial^2 U}{\partial y^2} \sim \frac{\nu U_s}{l^2} = \left[\frac{\nu}{ul}\frac{U_s}{u}\right]\frac{u^2}{l} \quad (6)$$

$$\frac{\partial \overline{u^2}}{\partial x} \sim \frac{u^2}{L} = \left[\frac{l}{L}\right]\frac{u^2}{l} \quad (7)$$

$$\frac{\partial \overline{uv}}{\partial y} \sim \frac{u^2}{l} = 1\cdot\frac{u^2}{l} \quad (8)$$

In the above equations, the mean streamwise velocity, U, has been normalized by the mean centerline velocity, U_s; the mean cross-stream velocity, V, by $U_s l/L$; the streamwise distance, x, by the distance from the orifice, L; cross-stream coordinate, y, by the jet half-width, l (defined as $U(l)/U_s = 0.5$); and the Reynolds stresses, $\overline{uv}$ or $\overline{u^2}$, by u^2. (Due to self-similarity, relation between some of the scales introduced above is possible.) The magnitude of pressure in Eq. (4) is not known, and therefore has been left as such (for conventional jets [17], $P \sim \rho u^2$; note that the same symbol P is used for both pressure and pressure scale).

In the limit of large Reynolds number, Re_l ($= \nu/ul$), the viscous terms Eqs. (5 and 6) can be dropped. Further, for conventional jets, Tennekes and Lumley [17] argue that since l/L is small, terms represented in Eqs. (4) and (7) can be neglected in Eq. (1), which leads to the following simplified equation for planar turbulent conventional jet:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial \overline{uv}}{\partial y} = 0. \quad (9)$$

However, based on the observation that the spread rate of synthetic jets is substantially larger than conventional jets, we refrain from making this latter simplification. For example, Smith and Glezer [1] report $l/L = 0.194$ for planar turbulent synthetic jet. Therefore, the following simplified equation is instead obtained for synthetic jet:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial \overline{u^2}}{\partial x} + \frac{\partial \overline{uv}}{\partial y} = 0. \quad (10)$$

While writing the above equation, it has been assumed that the terms in square brackets in Eqs. (2) and (3) are of order unity, which implies that $(u/U_s)^2 \sim l/L$, or, because l/L has been assumed to be of order unity, u/U_s is also of order unity. Note that the pressure term has been retained in the above equation because the scale of pressure is yet to be determined (we will however find that the pressure term can be dropped from the equation). In order to estimate the magnitude of pressure, the cross-stream momentum equation is considered.

The time-averaged cross-stream momentum equation can be written as

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + \frac{\partial \overline{uv}}{\partial x} + \frac{\partial \overline{v^2}}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right). \quad (11)$$

The first two terms in the above equation scale as $[(U_s/u)^2(l/L)^2]\cdot u^2/l$; the next two terms on the left scale as $(l/L)\cdot u^2/l$ and $1\cdot u^2/l$, respectively. The viscous terms on the right scale as $[(1/Re_l)(U_s l^3/uL^3)].u^2/l$ and $[(1/Re_l)(U_s l/uL)]\cdot u^2/l$, respectively; and the pressure term remains undetermined. As before, the viscous terms drop-off in the limit of large Reynolds number. For a conventional jet, under the assumption that l/L is small, $\partial\overline{uv}/\partial x$ can be neglected; further, because $(u/U_s)^2 \sim l/L$, the first two terms in the above equation also drop-off, leaving pressure to balance $\partial\overline{v^2}/\partial y$. The cross-stream momentum equation simplifies to $\partial\overline{v^2}/\partial y = -1/\rho\cdot \partial P/\partial y$ in the case of conventional jets, leading to $P \sim \rho u^2$ [17].

However, for synthetic jets, such simplification is not possible because both u/U_s and l/L are of order unity. Therefore, all the three non-linear terms – $U\partial V/\partial x$, $V\partial V/\partial y$ and $\partial\overline{uv}/\partial x$, being of the same order as $\partial\overline{v^2}/\partial y$, need to be retained. Hence, there is no requirement for pressure to balance $\partial\overline{v^2}/\partial y$, or pressure to scale as ρu^2. Rather, we believe that, pressure should not scale as ρu^2, else it would affect both the streamwise and cross-stream momentum equations Eqs. (10) and (11), which is unlikely for a jet discharged in an otherwise undisturbed surrounding [16,20].

The experiments of Smith and Glezer [1] suggest an important role for pressure near the orifice. They reported that pressure at the exit plane is proportional to r_0^{-2}, where r_0 is the radial distance from the orifice. Therefore, the flow experiences an adverse pressure gradient near the orifice. Although this gradient would persist in the self-similar regime, we do not expect pressure to significantly alter the momentum of the jet in the self-similar regime [5], and therefore neglecting pressure gradient in the momentum equations, for the self-similar regime, should be justified.

Therefore, finally, the cross-stream equation Eq. (11) for a planar synthetic jet simplifies to

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+\frac{\partial \overline{uv}}{\partial x}+\frac{\partial \overline{v^2}}{\partial y}=0 \tag{12}$$

and the streamwise equation Eq. (10) to

$$U\frac{\partial U}{\partial x}+\frac{\partial \overline{u^2}}{\partial x}+V\frac{\partial U}{\partial y}+\frac{\partial \overline{uv}}{\partial y}=0. \tag{13}$$

It can be easily verified that integrating Eq. (13) between the limits $y=-\infty$ to $y=+\infty$ with the help of continuity equation

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{14}$$

yields

$$\rho\int_{-\infty}^{+\infty}(U^2+\overline{u^2})\mathrm{d}y=M \tag{15}$$

where M is a constant. The last two terms in Eq. (13) drop-off upon integration because both U and $\overline{uv}$ tend to zero in the limit $y\to\pm\infty$. Note that M is not the total amount of momentum added at the orifice per unit depth per unit time by the diaphragm, rather less than it. The difference lies because of the adverse pressure gradient near the orifice mentioned above, which reduces the momentum in the self-similar region. Also note that Eq. (15) implies that the momentum of the jet calculated based on mean velocity alone will not be constant, rather the effect of streamwise velocity fluctuations also needs to be taken into account in the calculation of momentum.

Smith and Glezer [1] showed that the normalized momentum flux (obtained by integration of the fitted velocity profile) along the streamwise coordinate, is always less than unity. An asymptotic value of 0.55 was suggested by their experiments, which is consistent with Eq. (15) and the discussion following this equation. Further support for an asymptotically constant value of momentum flux is provided by the numerical simulations of Fugal et al. [16].

The above mentioned momentum equation Eq. (15) has been used by some researchers for continuous jets also (e.g. Ramaprian and Chandrasekhara [10]). The difference in the integral momentum equation of synthetic and continuous jet is therefore pointed out. To a first order effect, the momentum flux in continuous jets is calculated using mean streamwise velocity only [17,18]. Therefore, unlike their conventional counterparts, the velocity fluctuations play a more important role (and not merely appear as a second-order effect) in the case of synthetic jets.

2.2. Axisymmetric synthetic jet

The analysis for axisymmetric turbulent synthetic jet follows its planar counterpart. Starting with the time-averaged streamwise momentum equation [18]

$$U\frac{\partial U}{\partial x}+\frac{V}{r}\frac{\partial (rU)}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left[\frac{\partial^2 U}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)\right]-\frac{\partial \overline{u^2}}{\partial x}-\frac{1}{r}\frac{\partial (r\overline{uv})}{\partial r} \tag{16}$$

and performing an order an magnitude analysis for the various terms in the self-similar region, in a manner similar to that discussed above, we obtain:

$$U\frac{\partial U}{\partial x}+\frac{V}{r}\frac{\partial (rU)}{\partial r}+\frac{\partial \overline{u^2}}{\partial x}+\frac{1}{r}\frac{\partial (r\overline{uv})}{\partial r}=0. \tag{17}$$

In the above equations, r is the cross-stream coordinate. In reducing Eq. (16) to Eq. (17), again it has been assumed that both u/U_s and l/L are of order unity and the Reynolds number is large. The corresponding cross-stream momentum equation is

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial r}+\frac{\partial \overline{uv}}{\partial x}+\frac{\partial \overline{v^2}}{\partial r}+\frac{\overline{v^2}-\overline{w^2}}{r}=0, \tag{18}$$

where, w is the fluctuating component of azimuthal velocity. The integral form of momentum equation can be obtained by integrating Eq. (17) from $r=0$ to ∞.

3. Similarity solution

The streamwise variation of centerline velocity and jet half-width in the self-similar region are derived in this section. The analysis is first performed for planar synthetic jet, followed by its axisymmetric counterpart.

Let us assume

$$U=U_sf(\xi), \tag{19}$$

$$\overline{uv}=U_s^2h(\xi), \tag{20}$$

$$\overline{u^2}=U_s^2H(\xi), \tag{21}$$

where f, h and H are some functions (to be determined empirically) of ξ, and ξ is the normalized cross-stream coordinate ($\xi=y/l$ for planar case and $\xi=r/l$ for axisymmetric case). The cross-stream velocity can be determined from Eq. (19) and the continuity equation Eq. (14) as

$$V=-l\int_0^{\xi}\left(\frac{\mathrm{d}U_s}{\mathrm{d}x}f-\frac{U_s}{l}\xi\frac{\mathrm{d}f}{\mathrm{d}\xi}\frac{\mathrm{d}l}{\mathrm{d}x}\right)\mathrm{d}\xi. \tag{22}$$

On substituting Eqs. (19)–(22) in Eq. (13), and noting that both U_s and l are functions of x only, we obtain

$$\frac{l}{U_s}\frac{\mathrm{d}U_s}{\mathrm{d}x}f^2-\frac{\mathrm{d}l}{\mathrm{d}x}\xi\frac{\mathrm{d}f}{\mathrm{d}\xi}-\frac{l}{U_s}\frac{\mathrm{d}U_s}{\mathrm{d}x}\frac{\mathrm{d}f}{\mathrm{d}\xi}\int_0^{\xi}f\mathrm{d}\xi+\frac{\mathrm{d}l}{\mathrm{d}x}\frac{\mathrm{d}f}{\mathrm{d}\xi}\int_0^{\xi}\xi\frac{\mathrm{d}f}{\mathrm{d}\xi}\mathrm{d}\xi$$
$$+2\frac{l}{U_s}\frac{\mathrm{d}U_s}{\mathrm{d}x}H-\frac{\mathrm{d}l}{\mathrm{d}x}\xi\frac{\mathrm{d}H}{\mathrm{d}\xi}+\frac{\mathrm{d}h}{\mathrm{d}\xi}=0.$$

For self-similarity to hold, we require that the coefficients in the above equation should be constant, which yields

$$\frac{\mathrm{d}l}{\mathrm{d}x}=c, \tag{23}$$

$$\frac{l}{U_s}\frac{\mathrm{d}U_s}{\mathrm{d}x}=c_2, \tag{24}$$

where c and c_2 are constants (to be determined empirically). Note that these are the same conditions as for conventional jets [17]. While Eq. (23) suggests that $l\sim x$, Eq. (24) can be satisfied by any power law $U_s\sim x^n$. The value of n can be determined by invoking Eq. (15) along with Eqs. (19) and (21) to get $n=-1/2$. Therefore, both the mean and turbulent velocity scale should vary as $x^{-0.5}$.

The above analysis can be repeated with axisymmetric synthetic jets to obtain $l\sim x$ and $U_s\sim x^{-1}$, in agreement with their continuous counterparts [13]. The turbulent velocity scale should also vary as x^{-1} in the axisymmetric case.

4. Discussion

The results derived in the previous section are first validated and then the underlying assumptions in the analysis are justified by comparison against experimental results in the literature. Some extensions of the analysis are also discussed in this section.

4.1. Validation

Table 1 presents a comparison of streamwise variation of the centerline velocity and jet half-width from the present analysis with respect to the experimental data available in the literature. It can be seen that the predictions from the analysis compare well against experimental data, for both the planar and axisymmetric cases. The only deviation is for Smith and Glezer [1]; however, these authors reported that u scales as $x^{-0.5}$ which is in agreement with our analysis. Although the streamwise variation of turbulent velocity scale has not been documented in the table, u, wherever else reported, has been found to follow the same variation as U_s.

When comparing with conventional jets it should be kept in mind that, although the streamwise variation of velocity and length scales are similar between the two cases, the coefficients c and c_2 (in Eqs. (23) and (24)) may be quite different, making the flow different in appearance and also quantitatively. Such differences have been explored by Smith and Swift [15] among others. For example, Cater and Soria [5] report that the streamwise velocity for axisymmetric synthetic jet decays about seven times faster as compared to axisymmetric continuous jet. Hussein et al. [11] discuss the important role of initial condition in turbulent (continuous) jets. The mechanism of jet formation is entirely different between synthetic and continuous jets and differences should therefore not be unexpected.

We now examine the assumptions made in deriving the simplified form of momentum equation (Eq. (13) or (17)) with synthetic jet. Besides the assumption of large Reynolds number, the other primary assumptions are that both l/L and u/U_s are of order unity. Table 2 summarizes the values of some of the relevant parameters. The measurements of Smith and Glezer [1] are for $12 \leqslant x/d \leqslant 80$ and at a Reynolds number of 383. The data of Cater and Soria [5] has been collected between $x/d = 4$ to 74, at two different Reynolds numbers. Besides these, the measurements of Smith and Swift [15] suggest a spread rate of about 0.19 for planar synthetic jet. Similarly, the data of Mallinson et al. [14] suggest $\overline{u^2}/U_s^2 = 0.16$ at $x/d = 27$ for axisymmetric synthetic jet.

As is apparent from Table 2, the spread rate of synthetic jet is about twice of their continuous counterparts, at least in the planar case. Although it is probably acceptable to neglect terms of order l/L for continuous jets (as has been done, for example, by Tennekes and Lumley [17]), one should refrain from making such simplification for synthetic jets. Similarly, the data in the table suggests that u/U_s is at least 0.24 and 0.36 for planar and axisymmetric jets, respectively, and therefore, these two velocity scales can be regarded as being of order unity. The last two rows in Table 2 shows that $\overline{v^2}$ and $\overline{uv}$ are of the same order as $\overline{u^2}$, as assumed above. Therefore, the assumptions made in the analysis seem to be reasonable.

Table 1
Comparison of centerline velocity decay and jet half-width from various sources

Configuration of jet	Source	U_s	l	Comment
Planar synthetic	Smith and Glezer [1]	$x^{-0.58}$	$x^{0.88}$	Experimental
	Smith and Swift [15]	$x^{-0.5}$	x	Experimental
	Present	$x^{-0.5}$	x	Similarity analysis
Planar continuous	Tennekes and Lumley [17]	$x^{-0.5}$	x	Similarity analysis
Axisymmetric synthetic	James et al. [2]	x^{-1}	x	Experimental
	Mallinson et al. [14]	x^{-1}	x	Experimental
	Cater and Soria [5]	x^{-1}	x	Experimental
	Travnicek et al. [21]	x^{-1}	–	Experimental
	Present	x^{-1}	x	Similarity analysis
Axisymmetric continuous	Tennekes and Lumley [17]	x^{-1}	x	Similarity analysis

Table 2
Comparison of length and velocity scales, as obtained from experiments, for synthetic and continuous jets

	Planar		Axisymmetric	
	Synthetic [1]	Continuous [10]	Synthetic [5]	Continuous [12]
l/L	0.194	0.110	0.107	0.092
$\overline{u^2}/U_s^2$	0.07	0.04	0.31[a], 0.13[b]	0.068
$\overline{v^2}/U_s^2$	0.05	0.029	0.09[a], 0.07[b]	0.04
$\overline{uv}/U_s^2$	0.025	0.02	0.04[a], 0.02[b]	0.02

[a] Reynolds number = 1000 ($x/d = 30$).
[b] Reynolds number = 10000 ($x/d = 30$).

4.2. Further analysis

In a manner similar to Agrawal and Prasad [19], by assuming a Gaussian streamwise velocity profile (suggested, for example, by the experiments of Smith and Glezer [1], Cater and Soria [5]), i.e., using $f(\xi) = \exp(-\xi^2)$ in Eq. (19), an expression for V can be derived by employing Eq. (22), as

$$\frac{V}{U_s} = \frac{c}{4}\left(4\xi\exp(-\xi^2) - \sqrt{\pi}\mathrm{erf}(\xi)\right). \tag{25}$$

Similarly, for axisymmetric synthetic jet, we would obtain

$$\frac{V}{U_s} = \frac{c}{2\xi}\left(-1 + \exp(-\xi^2) + 2\xi^2\exp(-\xi^2)\right). \tag{26}$$

The interesting consequences of Eqs. (25) and (26), as first pointed out by Agrawal and Prasad [19], are: first, there is a change in sign for V at $\xi \approx 1.3$, which suggests an outflow near the centerline of the jet and inflow far away from it. The presence of outflow near the centerline is perhaps contrary to the notion of entrainment in turbulent jets. The reason for outflow is the decay of the centerline velocity (and not the assumption of a Gaussian velocity profile). Note that a bigger value of spread rate, c, implies a stronger outflow for synthetic jets as compared to their continuous counterparts. Second, care has to be taken while defining the entrainment velocity at the 'edge' of the jet, at least in the case of axisymmetric jet. This is apparent after plotting Eq. (26) which shows that V does not become constant for any value of ξ.

Agrawal and Prasad [19] went further to derive expressions for $\overline{uv}$ and turbulent eddy viscosity, using the simplified momentum Eq. (9). Such an analysis is however not possible in the present case unless the function H (in Eq. (21)) is explicitly specified.

5. Concluding remarks

A similarity analysis of planar and axisymmetric turbulent synthetic jets has been discussed in this paper, perhaps for the first time. Considering that, although self-similarity was seen from the experimental data about a decade back, no theoretical attempt has been made to derive the scaling relationships, our analysis should assume significance. The governing equations are first derived for flow in the self-similar regime; these equations are much more involved than for conventional jets revealing the complexity of synthetic jets. A similarity analysis is then performed and the streamwise variation of centerline velocity and jet half-width are obtained. The number of assumptions in the analysis is reasonably small and these have been justified from the experimental data in the literature. Interestingly, the analysis predicts the same streamwise variation for synthetic jets as their continuous counterparts. The coefficients are however different between the two cases, leading to different flow features. It is argued that the momentum flux in the self-similar region of synthetic jets should remain conserved, but with a value less than that supplied at the source.

The most important difference between continuous and synthetic jets arises due to a larger spread rate of synthetic jets. An analysis of the sort presented in the paper is expected to trigger further development on the understanding of synthetic jets. For example, the negligible role of pressure in the self-similar region conjectured here needs to be experimentally verified; similarly, the functional form of $\overline{u^2}$ should be determined to allow integration of the mean streamwise momentum equation. The results are significant because they tend to clearly bring out the similarities and differences of synthetic jets with respect to their continuous counterparts, the latter flows have been extensively studied in the literature.

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